# LDPC Codes Construction Based on 3D Cyclic Rectangular Lattice

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# Outline

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# Introduction

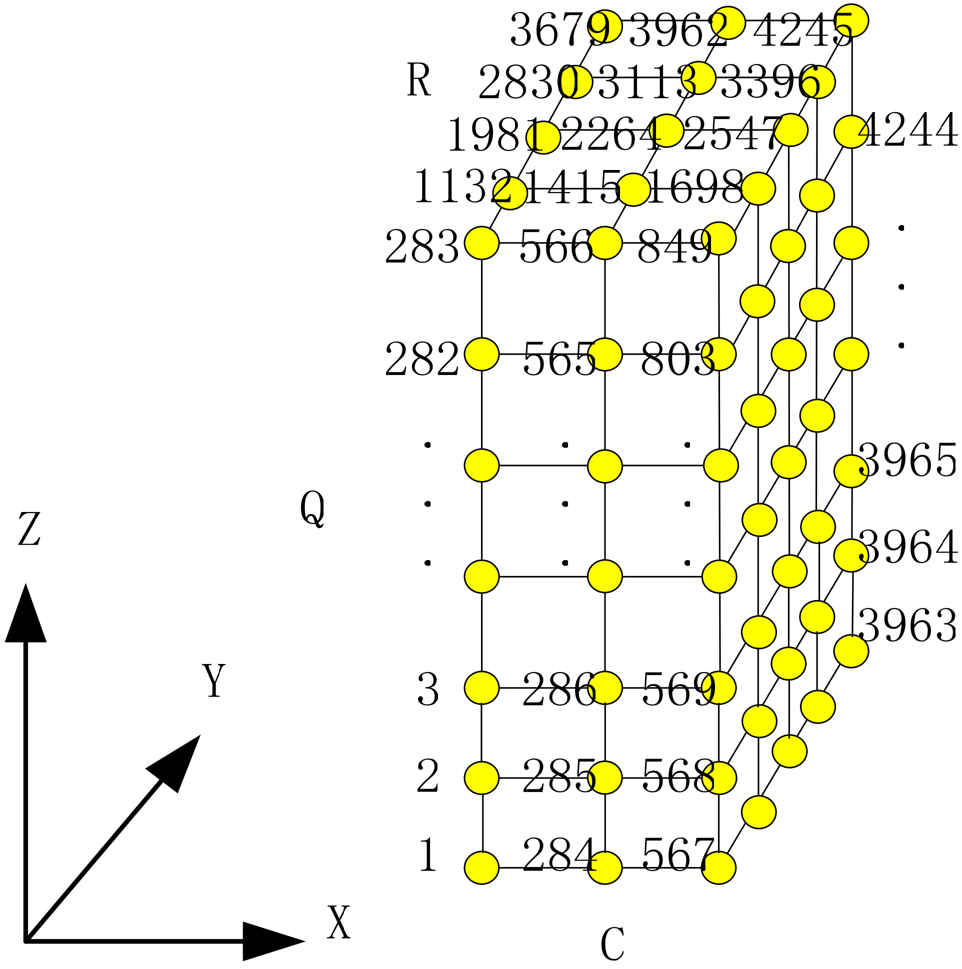
- Irregular LDPC codes have better performance,but lead to complicated hardware implementation because of the uncertainty of the node degree distribution.
- Gallager proved that the regular LDPC codes with column weight $j \geq 3$ have a minimum distance , that grows linearly with block length $N$ , but grows logarithmically with $N$ when $j=2$.
- Therefore we only consider the construction of ***regular*** LDPC codes with ***j=3***.

# 3D Cyclic Rectangular Lattice

- Vasic has presented a 2D lattice structure to generate BIBDs , we expand it to 3D cyclic rectangular lattice.
- Let the size of the lattice is $C \times R \times Q$, a 3D cyclic rectangular lattice can be denoted as:

$$Lattice(C,R,Q) = \{(x,y,z): 0 \le x \le C-1,$$
$$0 \le y \le R-1, 0 \le z \le Q-1\}$$

# An example of (3,5,283) 3D lattice



# Q-C planes construction in 3D lattice

- Construct some planes parallel with Z axis, if project them to the top face, these planes are mapped to some lines in the top face; Since the size of these planes is $Q \times C$, we will use the term "***Q-C planes***" to refer to them.
- So, to construct planes in the lattice which parallel with $Z$ axis means to construct lines on the top face which is an $R \times C$ 2-D lattice. These lines is determined by a start point $p_{rc} = (0, y_0, Q-1)$ and a slope $s_{rc}$ , denoted as follows

$$l(y_0, s_{rc}) = \{(x, y_0 + s_{rc} x \bmod(R), Q-1), 0 \le x \le C-1\}$$

- Lines on the top face can be denoted as $L=\{l(y_0,s_{rc}),0\le y_0\le R-1,0\le s_{rc}\le R-1\}$
- There are $R^2$ lines on the top face and each line contains $C$ points, i.e. there are $R^2$ $Q$-$C$ planes in the lattice and the size of each such plane is $Q\times C$. These planes can be represented as lines on the top face also. Therefore, a $Q$-$C$ plane can be denoted by $P_{qc}(y_0,s_{rc})$ .

# Lines Construction in Q-C Planes

- A line $l_{qc}$ on the Q-C plane $P_{qc}$ is determined by a start point $Z_0$ and a slope on the Q-C plane $s_{qc}$, denoted as

$$l_{qc}(z_0, s_{qc}) = l_{qc}(y_0, z_0, s_{qc})$$

$$= \{(x, y_0 + s_{rc} x \bmod(R), z_0 + s_{qc} x \bmod(Q)), 0 \le x \le C-1\}$$

where

$$0 \le y_0 \le R-1 \quad 0 \le z_0 \le Q-1$$

$$0 \le s_{rc} \le R-1 \quad 0 \le s_{qc} \le Q-1$$

- A line contains $C$ points and there are $Q^2R^2$ lines in the lattice.

- Points are defined as the elements on the lattice and blocks are defined as lines on different $Q$-$C$ planes and the points connected by a line is the elements of the block. It is obvious that this makes a $\{QRC, Q^2R^2, QR, C, 1\}$ BIBD and the design is free of 4-cycle naturally.

- In this case, the code length is $\boldsymbol{Q^2R^2}$, code rate is $\boldsymbol{1\text{-}C/QR}$.

- A 6-cycles means that there are some 3-subsets which contain three different elements and each duple of these three elements are included in the different block. So a 6-cycle can be represented as a 3-side polygon in the lattice structure. The 3-side polygons can be seen as triangles with blocks being the sides and lattice nodes the vertices.
- Similarly, the 8-cycle in the Tanner graph can be represented as a quadrangle.

# Girth-10 LDPC code construction

- To construct girth-10 LDPC codes, we should eliminate 6-cycles and 8-cycles in the Tanner graph;
- We select all *Q*-*C* planes and generate ***only one group parallel lines*** (the number is Q) in each plane with slope $s_{y_0, s_{rc}}$, where (0, $y_0$, *Q*-1) is the start point of the projection of the *Q*-*C* plane on the top face and $s_{rc}$ is the slope. So we can get BIBD-($QRC$, $QR^2$, $R$, $C$, 1).
- The code length in this case becomes $\boldsymbol{QR^2}$, and the code rate is $\boldsymbol{1-C/R}$.

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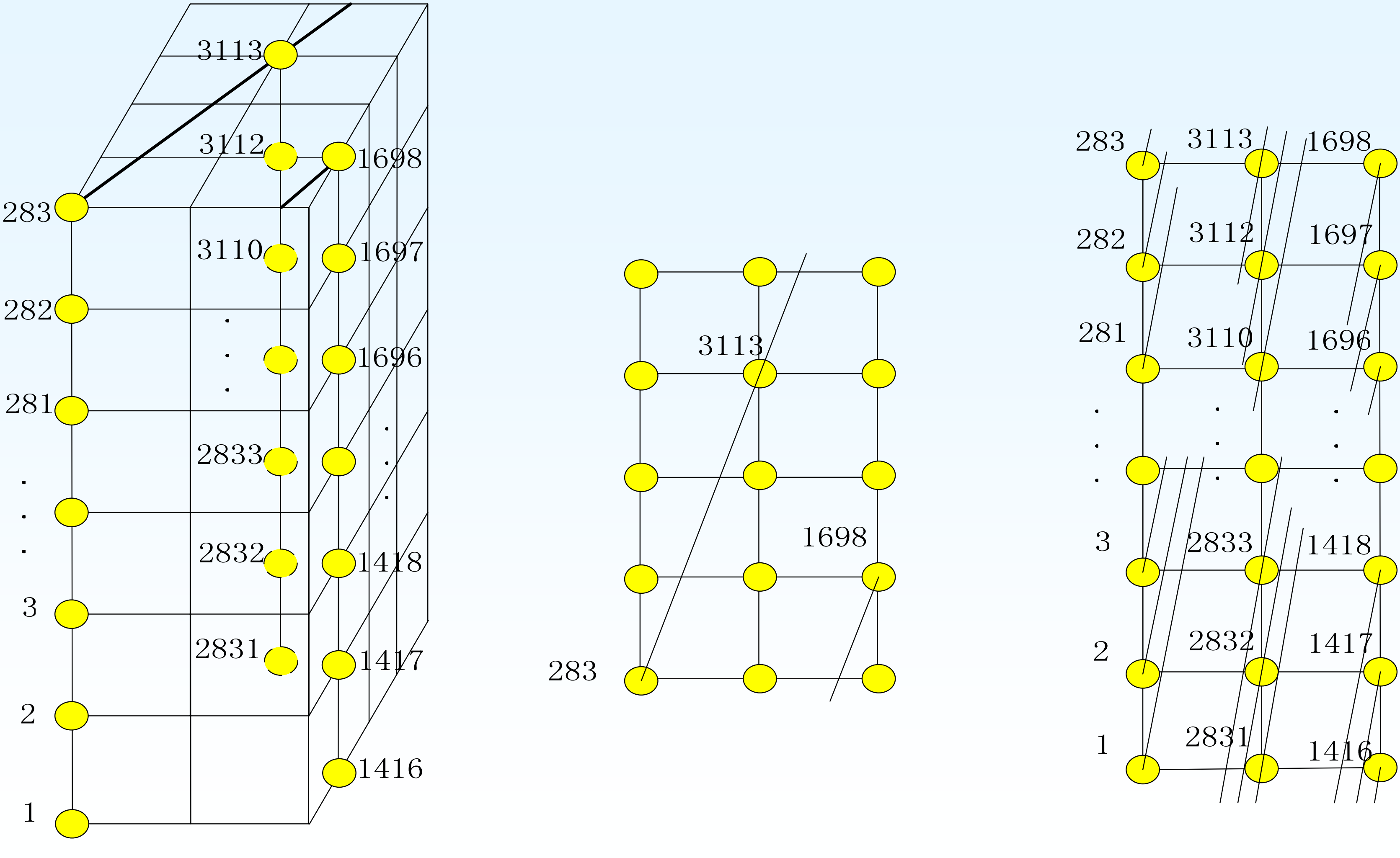


(a) (b) (c)

- An example of girth-10 LDPC codes (a) Lattice structure (b) Top face of the lattice and a line with $s_{rc}$=3. (c) The *Q-C* plane determined by the line of (b) and a group of lines with $S_{y_0,s_{rc}}$=63 .

# Some codes and their performance

| Q | R | C | N ( $Q^2R$ ) | M ($QRC$) | j | k | Code rate 1-$C/R$ | Drop point crossing BER=1e-4 (dB) | To Shannon limit (dB) |
|---|---|---|---|---|---|---|---|---|---|
| 45 | 5 | 3 | 1125 | 675 | 3 | 5 | 0.4 | 1.9 | 2.1 |
| 182 | 7 | 3 | 8918 | 3822 | 3 | 7 | 0.57 | 1.7 | 1.4 |
| 283 | 5 | 3 | 7075 | 4245 | 3 | 5 | 0.4 | 1.4 | 1.7 |
| 709 | 7 | 3 | 34741 | 14889 | 3 | 7 | 0.57 | 1.5 | 1.2 |
| 2099 | 11 | 3 | 253979 | 69267 | 3 | 11 | 0.73 | 2.2 | 0.91 |
| 3083 | 13 | 3 | 521027 | 120237 | 3 | 13 | 0.77 | 2.4 | 0.45 |

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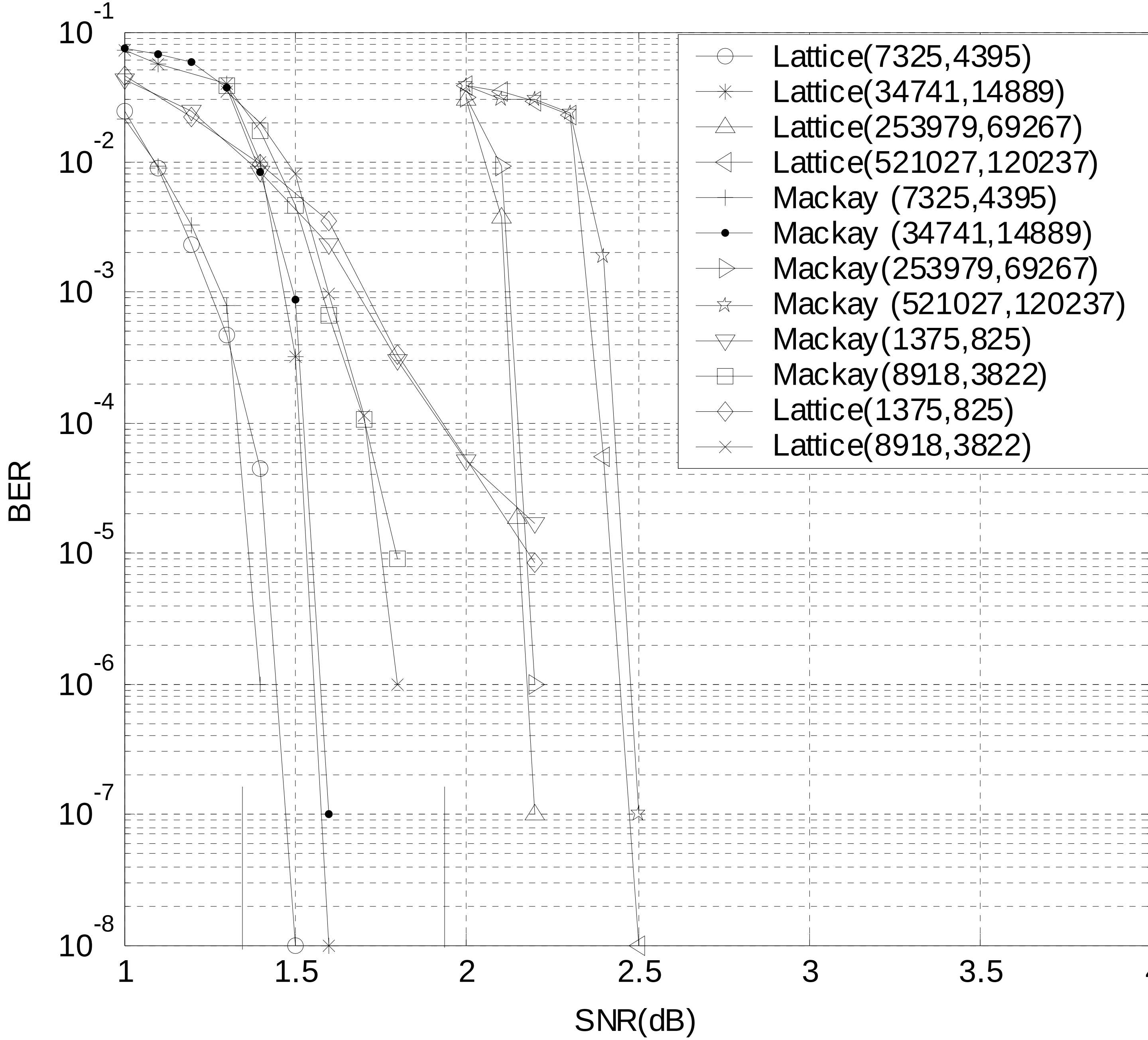

# Conclusion

- The main principle is to optimize the lines distribution in a 3-D cyclic rectangular lattice to avoid the forming of triangles and quadrangles.
- In this way, we get ($QRC$, $QR^2$, $R$, $C$, 1) BIBDs. Using the point-block incidence matrix as the parity-check matrix, we get a family of ($QR^2$, $C$, $QRC$) LDPC codes and the girth is at least 10.